



Unbounded Operators – Part 1

Linear Algebra

Real Analysis

Functional Analysis

Motivation:

- partial differential equations
- quantum mechanics: one needs operators X, P with

$$XP - PX = i \cdot I$$

Definition:

Let $(X, \|\cdot\|_X)$, $(Y, \|\cdot\|_Y)$ be normed spaces (same field $F \in \{\mathbb{R}, \mathbb{C}\}$) and $\mathcal{D} \subseteq X$ subspace.

A linear map $T: \mathcal{D} \rightarrow Y$ is called an operator.

Other notations:

- $T: X \supseteq \mathcal{D} \rightarrow Y$
- $T: X \rightarrow Y$ with domain $\mathcal{D}$
- $(T, \mathcal{D})$ or T with $\mathcal{D}(T) = \mathcal{D}$

Moreover: T is called densely defined if $\overline{\mathcal{D}}^{\|\cdot\|_X} = X$.

$$\text{Ran}(T) := \{Tx \mid x \in \mathcal{D}\} \subseteq Y \text{ subspace}$$

$$\text{Ker}(T) := \{x \in \mathcal{D} \mid Tx = 0\} \subseteq X \text{ subspace}$$

T is called bounded if $\exists C > 0 \forall x \in \mathcal{D} : \|Tx\|_Y \leq C \cdot \|x\|_X$

T is called unbounded if $\forall C > 0 \exists x \in \mathcal{D} : \|Tx\|_Y > C \cdot \|x\|_X$

Recall: T is bounded $\Leftrightarrow T$ is continuous at all points $x \in \mathcal{D}$

Therefore: T is unbounded $\Leftrightarrow T$ is not continuous (at no point $x \in \mathcal{D}$)