



PLU decomposition
 P → permutation matrix
 L → lower triangular
 U → row echelon form

$$P_{1 \leftrightarrow 2} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 0 & 1 & 2 & 1 & 2 \\ \boxed{1} & 0 & 0 & 0 & 1 \\ 2 & 1 & 2 & 1 & 5 \\ 1 & 2 & 4 & 3 & 6 \end{pmatrix} = P_{1 \leftrightarrow 2}^2 \begin{pmatrix} 0 & 1 & 2 & 1 & 2 \\ \boxed{1} & 0 & 0 & 0 & 1 \\ 2 & 1 & 2 & 1 & 5 \\ 1 & 2 & 4 & 3 & 6 \end{pmatrix}$$

$$= P_{1 \leftrightarrow 2} \begin{pmatrix} \boxed{1} & 0 & 0 & 0 & 1 \\ 0 & 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 & 5 \\ 1 & 2 & 4 & 3 & 6 \end{pmatrix}$$

$$= P_{1 \leftrightarrow 2} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \boxed{1} & 0 & 0 & 0 & 1 \\ \boxed{0} & 1 & 2 & 1 & 2 \\ \boxed{2} & 1 & 2 & 1 & 5 \\ \boxed{1} & 2 & 4 & 3 & 6 \end{pmatrix} \begin{matrix} \\ \text{II} - 0 \cdot \text{I} \\ \text{III} - 2 \cdot \text{I} \\ \text{IV} - 1 \cdot \text{I} \end{matrix}$$

$$= P_{1 \leftrightarrow 2} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \boxed{1} & 0 & 0 & 0 & 1 \\ 0 & \boxed{1} & 2 & 1 & 2 \\ 0 & \boxed{1} & 2 & 1 & 3 \\ 0 & \boxed{2} & 4 & 3 & 5 \end{pmatrix} \begin{matrix} \\ \\ \text{III} - 1 \cdot \text{II} \\ \text{IV} - 2 \cdot \text{II} \end{matrix}$$

$$= P_{1 \leftrightarrow 2} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{pmatrix} \begin{pmatrix} \boxed{1} & 0 & 0 & 0 & 1 \\ 0 & \boxed{1} & 2 & 1 & 2 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \boxed{1} & 1 \end{pmatrix}$$

$$= P_{1 \leftrightarrow 2} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{pmatrix} P_{3 \leftrightarrow 4}^2 \begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 2 & 1 & 2 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$= P_{1 \leftrightarrow 2} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{pmatrix} P_{3 \leftrightarrow 4} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 2 & 1 & 2 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= P_{1 \leftrightarrow 2} P_{3 \leftrightarrow 4} P_{3 \leftrightarrow 4} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{pmatrix} P_{3 \leftrightarrow 4} \begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 2 & 1 & 2 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \underbrace{P_{1 \leftrightarrow 2} P_{3 \leftrightarrow 4}}_P \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{pmatrix} \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 2 & 1 & 2 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}}_U$$