



PDE - Part 1 - Exercises

Exercise 1: Let $f: (0, \infty) \rightarrow \mathbb{R}$ be a continuously differentiable function.

We define $G: \mathbb{R}^n \setminus \{0\} \rightarrow \mathbb{R}$ by setting:

$$G(x) = f(\|x\|)$$

Euclidean norm on $\mathbb{R}^n$

Use the chain rule to determine $D^\alpha G$ for each multi-index with $|\alpha| = 1$.

Exercise 2: Consider $F: \mathbb{R}^3 \rightarrow \mathbb{R}$ given by $F(x) = (x_1^2 + x_2^2) \cdot \cos(x_3)$

and calculate $D^\alpha F$ for every multi-index with $|\alpha| = 2$.

Exercise 3: Consider the PDE $\partial_t u + x \partial_x u = x^2$ for a function $u(t, x)$ with $t \in \mathbb{R}$, $x \in \mathbb{R}$.

We want to find solutions that satisfy $u(0, x) = g(x)$ for all $x \in \mathbb{R}$ and a given function $g \in C^1(\mathbb{R})$.

(a) Assume that we have a solution $u(t, x)$.

What can you say about $u(t, \alpha(t))$ and a C^1 -function α ?

(b) Choose $\alpha(t)$ in a suitable way to construct a solution $u(t, x)$.