

Exercise 1. Product metrics

In the *Start Learning Logic* series you already learnt about the example $(A \rightarrow B) \Leftrightarrow (\neg B \rightarrow \neg A)$ of two logically equivalent statements. Show that the following statements are also logically equivalent by using truth tables:

1. $\neg(A \wedge \neg B) \Leftrightarrow (A \rightarrow B),$
2. $\neg(A \wedge B) \Leftrightarrow \neg A \vee \neg B.$

Exercise 2. Complement

One usually deals with subsets A, B , etc. of a given fixed set X . In such a situation it is useful to introduce $A^c := X \setminus A$ which is called the complement of A (with respect to (w.r.t.) the set X). Show for $A, B \subset X$

- (a) $A \setminus B = A \cap B^c,$
- (b) $(A \cap B)^c = A^c \cup B^c.$

Exercise 3. Cartesian Product

Let A, B and C be sets.

- (a) Show $A \times (B \cup C) = (A \times B) \cup (A \times C).$
- (b) Let $|A| = n$ and $|B| = m$ where $n, m \in \mathbb{N}$. Show that

$$|A \times B| = n \cdot m.$$

Exercise 4. Some functions

- (a) Consider the two functions $f_1 : \mathbb{R} \rightarrow \mathbb{R}, x \mapsto x^2$ and $f_2 : [0, \infty) \rightarrow \mathbb{R}, x \mapsto x^2$. For both functions calculate preimages of the sets $\{1\}$, $[4, 9)$ and $(-1, 0)$.

- (b) Consider the two functions $g_1 : \mathbb{R} \rightarrow [0, 1], x \mapsto |\sin(x)|$ and $g_2 : [0, 2\pi] \rightarrow \mathbb{R}, x \mapsto \sin(x)$. For both functions calculate images of the sets $(0, \pi/2)$, $[0, \pi)$ and $(0, 2\pi]$.

- (c) Consider the two functions $h_1 : \mathbb{R} \rightarrow \mathbb{R}$ and $h_2 : [-1, 1] \rightarrow [\sqrt{3}, 2]$ given by

$$x = (h_1(x) + 2)^2 - 2 \quad \text{and} \quad x^2 + h_2(x)^2 = 4.$$

Check whether h_1 and h_2 respectively are correctly defined.

- (d) Consider all 6 functions from above and find out which of them are injective, surjective and bijective. Try to provide proofs and counterexamples.