

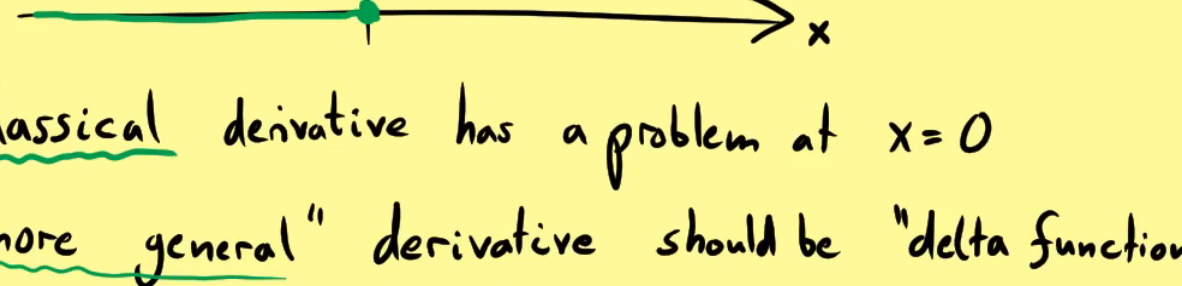
Distributions - Part 1

Analysis: Function, limit, derivative

Differential equations, Fourier series, Fourier transform

- ↳ solutions with sharp turns
- ↳ more general solutions

Historical example: 1927 Paul Dirac, $H: \mathbb{R} \rightarrow \mathbb{R}$ Heaviside function



- classical derivative has a problem at $x=0$

- "more general" derivative should be "delta function" δ !

$\delta(x) = 0$ for $x \neq 0$, and

$$\text{for } \varepsilon > 0: \int_{-\varepsilon}^{\varepsilon} \delta(x) dx = \int_{-\varepsilon}^{\varepsilon} H'(x) dx = H(\varepsilon) - H(-\varepsilon) = 1 - 0 = 1$$

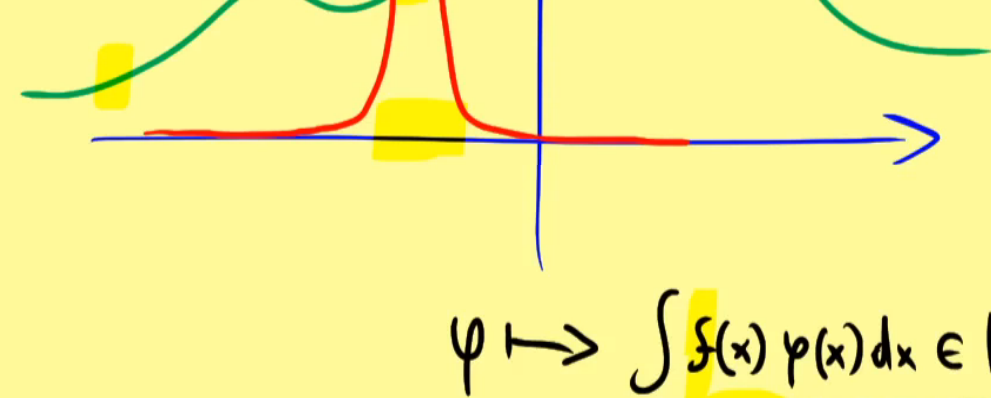
• δ can't be an ordinary function

$\delta(x) = 0$ almost everywhere (v.r.t. Lebesgue measure)

hence $\int_{-\varepsilon}^{\varepsilon} \delta(x) dx = 0 \neq 1$

• Dirac wants to calculate with $\delta', \delta'', \delta''' \dots$ → meaning?

$\Rightarrow \delta$ will be defined as a distribution → "generalised function"



Use so-called test functions

$\varphi: \mathbb{R} \rightarrow \mathbb{R}$

$$\varphi \mapsto \int_{\mathbb{R}} \delta(x) \varphi(x) dx \in \mathbb{R}$$

Linear map

$$\varphi \mapsto \varphi(0)$$

Delta function:

